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A NEW METHOD OF THE CALCULATION OF THE TWO-NEUTRINO DOUBLE BETA DECAY AMPLITUDES

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A new method is proposed of the calculation of the two-neutrino double-beta decay amplitude, which does not use the spectrum of intermediate nuclear states. The method is based on evaluation of a series of commutators of the nuclear Hamiltonian and weak nuclear hadron current. As a result, two-neutrino double-beta decay amplitude with a nuclear matrix element of a simple form of the two-nucleon operator has been obtained.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Новый метод вычисления амплитуды двухнейтринного двойного бета-распада

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Предложен новый метод вычисления скорости двухнейтринного двойного бета-распада, основанный на коммутационных соотношениях ядерного гамильтониана и слабых ядерных адронных токов. Метод не требует построения спектра "промежуточного" ядра. Получено замкнутое выражение для амплитуды данного процесса.

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Introduction

Recent first experimental observation of the two-neutrino mode of the double beta decay of ^{82}Se /1/ has revived interest in this process both among theoreticians and experimentalists.

The two-neutrino emitting mode of the double beta decay ($2\nu 2\beta$):

$$(A,Z) \rightarrow (A,Z+2) + 2e^- + 2\tilde{\nu}_e \, , \qquad (1)$$

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occurs as a second-order weak interaction process within the standard model of electroweak interactions.

In the calculations of the $(2\nu 2\beta)$ decay rate the two-nucleon mechanism is considered most frequently [2-4].

The modern method of the calculations of the $(2\nu 2\beta)$ amplitude as used recently requires the construction of the spectrum of intermediate nuclear states [2-4].

The early calculations by the closure approximation method have systematically overestimated the $(2\nu 2\beta)$ amplitude [2-4]. The recent progress in calculations of the $(2\nu 2\beta)$ amplitude has been achieved by using quasiparticle random-phase approximation method [5-8] which leads to a strong suppression of the $(2\nu 2\beta)$ nuclear matrix elements. However, the value of the nuclear matrix element is very sensitive to the particle-particle interaction in the spin-isospin polarisation force.

This substantial sensitivity to the details of the nuclear model used in the construction of intermediate nuclear states showed the importance of the elaboration of alternative methods of the calculations of the $(2\nu 2\beta)$ amplitude.

Recently, C.R.Ching and T.H.Ho [9] have proposed an alternative method for the calculations of the $(2\nu 2\beta)$ amplitude in which the sum over the intermediate nuclear states has been transformed into a series of commutators of two axial vector currents and the nuclear Hamiltonian.

In this article, we present another method which is also based on calculations of commutators of the nuclear Hamiltonian and weak nuclear currents. Performing summation of a series of commutators explicitly, we were able to derive the $(2\nu 2\beta)$ nuclear metrix element in a simple form of the two-nucleon operator for a given nuclear Hamiltonian.

The derivation of the $(2\nu 2\beta)$ amplitude

We assume that the beta decay Hamiltonian has the form

$$\mathcal{H}^{\beta} = \frac{G_F}{\sqrt{2}} 2(\bar{e}_L \gamma_\alpha \nu_{eL}) j_\alpha + h.c., \qquad (2)$$

where j_α is the strangeness conserving charged hadron current and e_L and ν_{eL} are operators of the left components of fields of the electron and neutrino, respectively.

$$<f|S^{(2)}|i> = \frac{(-i)^2}{2}\left(\frac{G_F}{\sqrt{2}}\right)^2 \frac{1}{(2\pi)^6} \frac{1}{\sqrt{16 p_{10} p_{20} k_{10} k_{20}}} \times$$

$$\times \bar{u}(p_1)\gamma_\alpha(1+\gamma_5)u^c(k_1)\cdot\bar{u}(p_2)\gamma_\beta(1+\gamma_5)u^c(k_2) J_{\alpha\beta} + \tag{3}$$

$$-(p_1 \rightleftarrows p_2) - (k_1 \rightleftarrows k_2) + (p_1 \rightleftarrows p_2)(k_1 \rightleftarrows k_2),$$

where,

$$J_{\alpha\beta} = \int e^{-i(p_1+k_1)x_1} e^{-i(p_2+k_2)x_2} \times$$

$$\times <p_f|T(J_\alpha(x_1) J_\beta(x_2))|p_i> dx_1 dx_2. \tag{4}$$

Here, p_1 and p_2 (k_1 and k_2) are four-momenta of the electrons (antineutrinos), p_i and p_f are four momenta of the initial and final nucleus, and $J_\alpha(x)$ is the weak charged nuclear current in the Heisenberg representation.

If we use the definition of time-order product of two operators in the form

$$T(J_\alpha(x_1) J_\beta(x_2)) = J_\alpha(x_1) J_\beta(x_2) + \theta(x_{20}-x_{10})[J_\beta(x_2), J_\alpha(x_1)], \tag{5}$$

we obtain

$$J_{\alpha\beta} = \sum_n 2\pi\delta(E_f - E_n + p_{10} + k_{10}) 2\pi\delta(E_n - E_i + p_{20} + k_{20}) \times$$

$$\times \int e^{-i(\vec{p}_1+\vec{k}_1)\cdot\vec{x}_1} e^{-i(\vec{p}_2+\vec{k}_2)\cdot\vec{x}_2} \times$$

$$\times <p_f|J_\alpha(0,\vec{x}_1)|p_n><p_n|J_\beta(0,\vec{x}_2)|p_i> d\vec{x}_1 d\vec{x}_2 + \tag{6}$$

$$+ 2\pi\delta(E_f - E_i + p_{10} + k_{10} + p_{20} + k_{20}) \times$$

$$\times \int e^{-i(\vec{p}_1+\vec{k}_1)\cdot\vec{x}_1} e^{-i(\vec{p}_2+\vec{k}_2)\cdot\vec{x}_2} \int_0^\infty e^{i(p_{20}+k_{20})t} \times$$

$$\times <p_f|[J_\beta(t,\vec{x}_2), J_\alpha(0,\vec{x}_1)]|p_i> \times dt\, d\vec{x}_1 d\vec{x}_2.$$

Here, $|p_n>$ is an eigenvector of an intermediate nucleus with energy E_n, and E_i and E_f are energies of the initial and final nucleus, respectively.

We can see that the first term in the r.h.s. of eq. (6) corresponds to two subsequent nuclear beta decay processes. However, in the case of the double beta decay, the beta transition from the parent nucleus (A, Z) to the intermediate nucleus $(A, Z+1)$ is forbidden energetically $(E_n > E_i)$ which implies that the first term in the r.h.s. of eq. (6) is equal to zero. So, only the second term in eq. (6) with the non-equal-time commutator of the nuclear hadron currents contributes to the $(2\nu 2\beta)$ amplitude.

Next, if we consider the non-relativistic impulse approximation for the hadronic currents,

$$J_\alpha(0,\vec{x}) = \sum_n \tau_n^+ (\delta_{\alpha 4} + i g_A \delta_{\alpha k} (\vec{\sigma}_n)_k)\, \delta(\vec{x} - \vec{x}_n) , \qquad (7)$$

limit the consideration only to $s_{1/2}$ wave states of the emitted electrons and antineutrinos and the most energetically favoured $0^+ \to 0^+$ nuclear transition, then for $J_{\alpha\beta}$ we can write,

$$J_{\alpha\beta} = 2\pi\delta(E_f - E_i + p_{10} + k_{10} + p_{20} + k_{20}) \times$$

$$\times \int_0^\infty e^{i(p_{20}+k_{20})t} \{\delta_{\alpha 4}\, \delta_{\beta 4} < p_f(0^+) | [e^{iHt} V(0) e^{-iHt}, V(0)] | p_i(0^+) > +$$

$$- \frac{1}{3} g_A^2 \delta_{\alpha k}\, \delta_{\beta k} < p_f(0^+) | [e^{iHt} A_\ell(0) e^{-iHt}, A_\ell(0)] | p_i(0^+) > \} dt$$

with

$$V(0) = \sum_n \tau_n^+ , \qquad (9)$$

$$A_\ell(0) = \sum_n \tau_n^+ (\vec{\sigma}_n)_\ell , \qquad (10)$$

where H is the nuclear Hamiltonian. If we use, instead of eq. (5), the fully equivalent formula for the time-ordered product of two operators,

$$T(J_\alpha(x_1) J_\beta(x_2)) = J_\beta(x_2) J_\alpha(x_1) + \theta(x_{10} - x_{20})[J_\alpha(x_1), J_\beta(x_2)] , \qquad (11)$$

we obtain the same formula for $J_{\alpha\beta}$ as in eq. (8) but with $p_{20} + k_{20}$ to be replaced by $p_{10} + k_{10}$.

From the equivalence of both the ways of calculation it follows that in the framework of our approximations $J_{\alpha\beta}$ does not depend on the kunematical variables and we can set

$$p_{10} + k_{10} = p_{20} + k_{20} = \frac{E_i - E_f}{2} = \Delta \,, \tag{12}$$

in eq. (8). The traditional way for the calculation of the $(2\nu 2\beta)$amplitude is to insert the complete set of intermediate nuclear states into the nuclear matrix elements in eq. (8).

Instead, we proceed as follows. We assume the following form of the nuclear Hamiltonian

$$H = H_0 + V_s \,, \tag{13}$$

where H_0 is the Hamiltonian of A free nucleons, and V_S is the effective nucleon-nucleon strong interaction

$$V_S = V_W + V_B + V_H + V_M \,, \tag{14}$$

where

$$V_W = \frac{1}{2} \sum_{i \neq j} g_W(r_{ij}) \,, \tag{15}$$

$$V_B = \frac{1}{2} \sum_{i \neq j} g_B(r_{ij}) \, P_\sigma \,, \tag{16}$$

$$V_H = \frac{1}{2} \sum_{i \neq j} g_H(r_{ij}) \, P_\tau \,, \tag{17}$$

$$V_M = \frac{1}{2} \sum_{i \neq j} g_M(r_{ij}) \, P_\sigma P_\tau \,, \tag{18}$$

with

$$P_\sigma = \frac{1}{2}(1 + \vec{\sigma}_i \cdot \vec{\sigma}_j) \,, \tag{19}$$

$$P_\tau = \frac{1}{2}(1 + \vec{\tau}_i \cdot \vec{\tau}_j) \,, \tag{20}$$

where $g_W(r_{ij})$, $g_B(r_{ij})$, $g_H(r_{ij})$ and $g_M(r_{ij})$ are scalar functions of the relative coordinate r_{ij} of two nucleons (i,j) /10/.

The tensor force, spin-orbit interaction and other nonlocal forces as well as the Coulomb interaction are neglected since their contributions are much smaller than those of V_B, V_H and V_M.

The nuclear matrix element with axial current is

$$M_{AA} = \langle p_f\ (0^+)\,|[\,e^{iHt}\ A_\ell(0)\ e^{-iHt}\ , A_\ell(0)]\,|\,p_i\,(0^+)\rangle\ . \qquad (21)$$

If we take into account that

$$[A_\ell(0)\,,\, H_0 + V_W] = 0\ , \qquad (22)$$

and if we suppose that the main contribution to the nuclear matrix element gives the two-nucleon operators and neglect the contribution of the three- and more-nucleon operators, we obtain

$$M_{AA} = \langle p_f\,(0^+)\,|\,\frac{1}{2}\sum_{n\neq m}[\,e^{iG_{nm}t}\ (\vec{A}_{nm})_\ell\ e^{-iG_{nm}t}\ , (\vec{A}_{nm})_\ell\,]|p_i(0^+)\rangle \qquad (23)$$

where

$$G_{nm} = g_B\,(r_{nm})\,P_\sigma + g_H(r_{nm})\ P_\tau + g_M(r_{nm})\,P_\sigma\,P_\tau\ , \qquad (24)$$

$$\vec{A}_{nm} = \tau_n^+\vec{\sigma}_n + \tau_m^+\vec{\sigma}_m\ . \qquad (25)$$

The two-nucleon operator of the nuclear matrix element in eq. (23) can be simplified in the following way. We have

$$e^{iG_{nm}t}\,\vec{A}_{nm}\,e^{-iG_{nm}t} = \qquad (26)$$
$$= e^{ig_M P_\sigma P_\tau t}\ e^{ig_H P_\tau t}\ e^{ig_B P_\sigma t}\ \vec{A}_{nm}\,e^{-ig_B P_\sigma t}\ e^{-ig_H P_\tau t}\ e^{-ig_M P_\sigma P_\tau t}\ ,$$

where, $g_M = g_M(r_{nm})$, $g_H = g_H\,(r_{nm})$, $g_B = g_B\,(r_{nm})$, and we can write /11/ that

$$e^{ig_B P_\sigma t}\ \vec{A}_{nm}e^{-ig_B P_\sigma t} = \sum_{k=0}^{\infty}\frac{(ig_B t)^k}{k!}\,[\,\overbrace{P_\sigma\ \ldots[\,P_\sigma}^{k}\ ,\vec{A}_{nm}]\ldots]\,. \qquad (27)$$

It is easy to see that using $P_\sigma^2 = 1$ we can sum up the series of commutators in eq. (27) and obtain

$$e^{ig_B P_\sigma t}\ \vec{A}_{nm}e^{-ig_B P_\sigma t} = \frac{1}{2}\{\vec{A}_{nm} + P_\sigma\,\vec{A}_{nm}P_\sigma\ + $$
$$+\cos(2g_B t)(\vec{A}_{nm} - P_\sigma\,\vec{A}_{nm}P_\sigma\ ) + \qquad (28)$$
$$+\,i\,\sin(2g_B t)\,[\,P_\sigma\ ,\vec{A}_{nm}\,]\}\,.$$

Proceeding similarly for the Heisenberg force $g_H(r_{nm})$ and for the Majorana force $g_M(r_{nm})$ is eq. (26) and then inserting the result into eq. (23) and calculating the commutator in eq. (23) with the use of the form of $\vec{A}_{nm}$ in eq. (25), we obtain

$$M_{AA} = 6i < p_f(0^+) | \sum_{n,m} \tau_n^+ \tau_m^+ \{ \sin(2(g_H - g_B)t)\, \Pi_s^\sigma + \frac{1}{3} \sin(2(g_H + g_B)t)\, \Pi_t^\sigma \} | p_i(0^+) > , \tag{29}$$

where

$$\Pi_s^\sigma = \frac{1}{2}(1 - P_\sigma) , \tag{30}$$

$$\Pi_t^\sigma = \frac{1}{2}(1 + P_\sigma) , \tag{31}$$

are projection operators which project onto the singlet (5) and triplet (t) parts of the nuclear two-body wave function.

Analogously, we get

$$M_{VV} = < p_f(0^+) | [e^{iHt} V(0) e^{-iHt}, V(0)] | p_i(0^+) > = 0 . \tag{32}$$

By inserting eqs. (29) and (32) into eq. (8) and then into eq. (3) and performing the integration over time variable using the standard procedure of adiabatic switching-off the interaction at $t \to \infty$,

$$\int_0^\infty e^{iat} \sin(bt)\, dt \Rightarrow \lim_{\epsilon \to 0} \int_0^\infty e^{i(a+i\epsilon)t} \sin(bt)\, dt = \lim_{\epsilon \to 0} \frac{b}{b^2 - a^2 - i\epsilon} , \tag{33}$$

we obtain the $(2\nu 2\beta)$ amplitude in the form

$$<f| S^{(2)} |i> = i \left(\frac{G_F}{\sqrt{2}}\right)^2 \frac{1}{(2\pi)^6} \frac{1}{\sqrt{16 p_{10} p_{20} k_{10} k_{20}}} \times$$

$$\times \{ \bar{u}(p_1) \gamma_k (1+\gamma_5) u^c(k_1) \cdot \bar{u}(p_2) \gamma_k (1+\gamma_5) u^c(k_2) - (k_1 \rightleftarrows k_2) \} \times \tag{34}$$

$$\times g_A^2 M^{2\nu 2\beta} 2\pi \delta(E_f - E_i + p_{10} + k_{10} + p_{20} + k_{20}) ,$$

where

$$M^{2\nu 2\beta} = \langle p_f(0^+)| \sum_{n\neq m} \tau_n^+ \tau_m^+ (f^-(r_{nm}) \Pi_s^{\sigma} + \frac{1}{3} f^+(r_{nm}) \Pi_t^{\sigma}) | p_i(0^+) \rangle, \quad (35)$$

with

$$f^{\pm}(r_{nm}) = \lim_{\epsilon \to 0} \frac{g_H(r_{nm}) \pm g_B(r_{nm})}{(g_H(r_{nm}) \pm g_B(r_{nm}))^2 - (\frac{\Delta}{2})^2 - i\epsilon}. \quad (36)$$

Discussion and conclusion

We have derived a new formula of the $(2\nu 2\beta)$ amplitude (eq.(34)) with the $(2\nu 2\beta)$ nuclear matrix element in which the summation of a series of commutators of the nuclear Hamiltonian and weak nuclear currents has been performed and a simple form of two-nucleon transition operator has been found.

The two-nucleon operator of the $(2\nu 2\beta)$ nuclear matrix element in eq. (35) in the framework of our approximations depends only on the Bartlett and Heisenberg forces and has a pole structure in their radial dependence (eq. (36)).

We note that if in the L-S coupling we consider only the configuration $L=0$, $S=0$ in the ground states of even-even nuclei and neglect the $L=1$ $S=1$ configuration, becasue of a short range of pairing forces /9/, and choose the Bartlett and Heisenberg forces to be equal, we obtain that the $(2\nu 2\beta)$ amplitude is equal to zero, which is in agreement with the conclusions of C.R.Ching and T.H.Ho /9/.

In conclusion, we add that the same method can be applied to the calculations of other nuclear processes such as the neutrinoless double-beta decay and nuclear pion double charge exchange.

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Note: After the present research has been completed, we have seen the paper by C.R.Ching, T.H.Ho and X.R.Wu /12/. The result obtained by those authors in a completely different way fully supports our result. It is, however, not clear from the text whether the pole structure present in eq. (36) has been noticed in Ref. /12/ .

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